

zan et al. (1968) and Potter and Graber (1972), used equation (1), which is only true for the primary flow, in the calculation of the perturbation equations. Actually, for the real flow, the temperature drop across the film is no longer linear:

$$\hat{T} = T_0 + (T_1 - T_0)\eta + T' \quad (2)$$

where T' is the perturbation quantity and $\bar{T} = T_0 + (T_1 - T_0)\eta$ the temperature of the primary flow.

The viscosity has first a temperature dependence rather than a film thickness dependence; hence the perturbation quantity T' induces a viscosity perturbation term which has to be taken into account to obtain the Orr-Sommerfeld equation. With the same assumptions as used by Shair (1971), the viscosity term is:

$$\mu = \mu_0 e^{-\alpha\eta - \alpha t'} \quad (3)$$

where t' is the dimensionless temperature perturbation term.

Linearization of equation (3) yields

$$\mu = \mu_0 e^{-\alpha\eta} (1 - \alpha t') \quad (4)$$

When the viscosity perturbation term is not omitted, the Orr-Sommerfeld equation reads:

$$\begin{aligned} i k R [(\bar{U} - C)(\phi'' - k^2\phi) - \bar{U}''\phi] \\ = \tilde{\mu} [\phi^{IV} - 2k^2\phi'' + k^4\phi] \\ + \frac{\tilde{d}\mu}{d\eta} [2(\phi''' - k^2\phi')] \\ + \bar{U}'(Z'' + k^2Z) + 2\bar{U}''Z'] \\ + \frac{\tilde{d}^2\mu}{d\eta^2} [\phi'' + k^2\phi + 2(\bar{U}'Z' \\ + \bar{U}''Z)] + \frac{\tilde{d}^3\mu}{d\eta^3} \bar{U}'Z \end{aligned} \quad (5)$$

where Z is defined as:

$$t' = Z(\eta) \exp [ik(x - c\tau)] \quad (6)$$

This modified Orr-Sommerfeld equation includes supplementary term of function Z , which do not exist in equation (9) of Yih and Seagrave. If the function Z appears in the Orr-Sommerfeld equation, one has to take into account the energy balance equation which yields:

$$i k Pe [(\bar{U} - c)Z - \phi] = Z'' - k^2Z \quad (7)$$

where Pe is the Peclet number defined as

$$Pe = \frac{\Delta \langle U \rangle \rho_1 C_p \Delta}{\lambda} \quad (8)$$

(C_p : heat capacity; λ : heat conductivity).

The additional boundary conditions read:

assuming a constant wall temperature:

$$Z(0) = 0 \quad (9)$$

assuming a constant interfacial temperature:

$$Z(1) + a = 0 \quad (10)$$

The two Orr-Sommerfeld type equations (5 and 7) have now to be solved, with their boundary conditions. The method used by Yih and Seagrave does no longer apply in this case.

However, the method of quadrature by differentiation developed by Solesio (1977) can be used; this method involves development of ϕ and Z at $\eta = 0$ and $\eta = 1$, and the viscosity terms (equation 1) are to be linearized at these same points.

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To the Editor:

The modification of the viscosity-temperature relationship suggested by Spindler to include the effect of perturbations in temperature is an interesting idea about which we would make the following comments:

1. The relation

$$\mu = \mu_0 e^{-\alpha\eta - \alpha t'} \quad (1)$$

is itself an approximation valid only in the case where $T_1 - T_0/T_0^2$ and T'/T_0 are much less than unity. This is easily shown by substituting

$$T = T_0 + (T_1 - T_0)\eta + T' \quad (2)$$

into the viscosity temperature relation

$$\mu = \mu_0 e^{-Ea(1/T_0 - 1/T)} \quad (3)$$

and expanding, to show that $t' = Ea \frac{T'}{T_0^2}$. When T'/T_0 is much less than

unity, the value of t^2 is correspondingly smaller.

Unquestionably the inclusion of the term $e^{-\alpha t'}$ contributes to additional terms in the Orr-Sommerfeld equation. We wrote the energy equation corresponding to Spindler's Equation (7), which in our nomenclature is

$$\hat{F}'' - k^2 \hat{F} = i k Pe [(\bar{U} - c)\hat{F} + \phi]$$

but did not find it necessary to include it in our paper.

3. In our opinion, the simplifications needed to justify Equation (1) do not significantly expand the scope under which the same relationship without the perturbation term is valid. The added mathematical complexity will probably not be warranted by any significant improvement in the results. Our treatment is preferable from an analytical point of view and is consistent with developments by Wazzan et al. (1968) and Potter and Graeber (1972).

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ERRATUM

In "Equilibrium and Kinetic Studies of Hydrogen Isotope Exchange on Vanadium Hydride," by Y. W. Wong and F. B. Hill [*AIChE J.*, **25**, 592 (1979)], the void fraction \bullet should not appear in Equation (24). Correction of this error leads to an average value of the tortuosity factor of 2.23 ± 0.36 . This value is not inconsistent with the notion that axial dispersion is due solely to molecular diffusion. In the Notation list, u should be defined as the actual linear gas velocity.